CHAPTER FIVE

AC STEADY STATE ANALYSIS

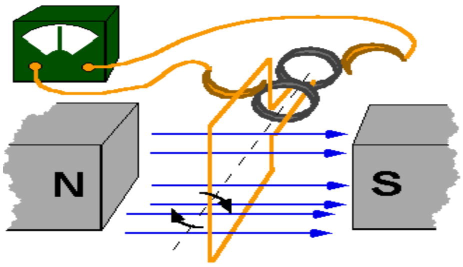
5.1 Introduction to Sinusoids

Introduction to Ac generator

Electricity is produced by generators at power stations and then distributed by a vast network of transmission lines (called the National grid system) to industry and for domestic use. It is easier and cheaper to generate alternating current (AC) than direct current (DC) and ac is more conveniently distributed than dc. Since its voltage can be readily altered using transformers.

One way to generate an AC voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field. The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (Faraday's law), and its polarity is dependent on the direction the coil sides move through the field. Since the rate of cutting flux varies with time, the resulting voltage will also vary with time.

- A sinusoid is a signal that has the form of the sine or cosine function.
- A sinusoidal current is usually referred to as alternating current (AC). Such a current reverses at regular time intervals and has alternately positive and negative values. Circuits driven by sinusoidal current or voltage sources are called AC circuits.



- Since the rate of cutting flux varies with time, the resulting voltage will also vary with time.
- The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (Faraday's law), and its polarity is dependent on the direction the coil sides move through the field.

For example in Figure 5.1 in (a), since the coil sides are moving parallel to the field, no flux lines are being cut and the induced volt- age at this instant (and hence the current) is zero. (This is defined as the 0^0 position of the coil.) As the coil rotates from the 0^0 position, coil sides AA' and BB' cut across flux lines; hence, voltage builds, reaching a peak when flux is cut at the maximum rate in the 90^0position as in (b). Note the polarity of the voltage and the direction of current. As the coil rotates further, voltage decreases, reaching zero at the 180^0 position when the coil sides again move parallel to the field as in (c). At this point, the coil has gone through a half-revolution.

During the second half-revolution, coil sides cut flux in directions opposite to that which they did in the first half revolution; hence, the polarity of the induced voltage reverses. As indicated in (d), voltage reaches a peak at the 270^0 point, and, since the polarity of the voltage has changed, so has the direction of current. When the coil reaches the 360^0 position, voltage is again zero and the cycle starts over. Figure 5.2 shows one cycle of the resulting wave form. Since the coil rotates continuously, the voltage produced will be a repetitive, periodic wave form (a waveform that continually repeats itself after the same time interval).

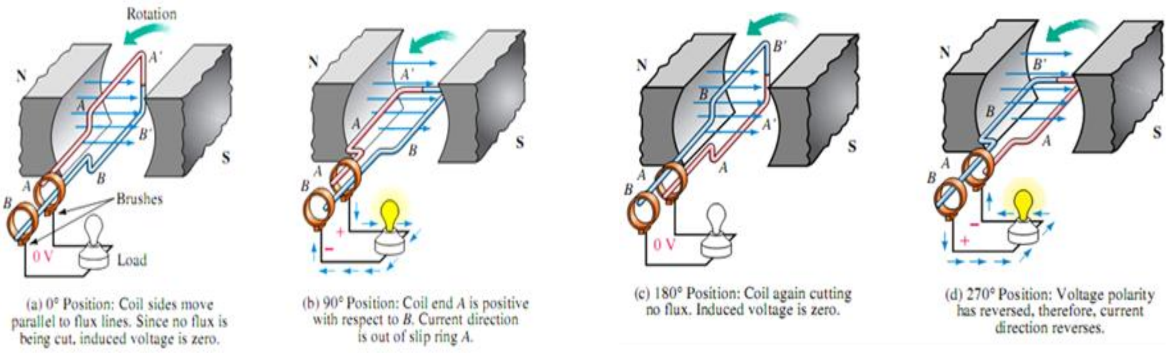


Fig.5. 1: Generating an AC voltage and Coil voltage versus angular position

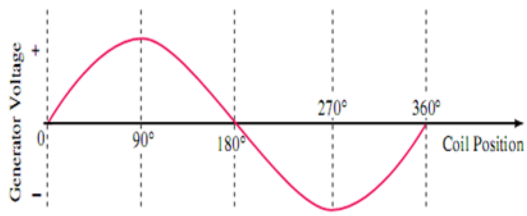
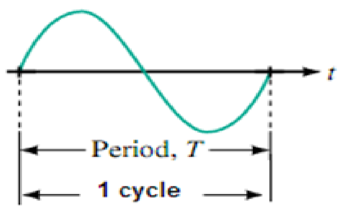


Fig.5. 2: one cycle of the resulting wave form

Period (T): the time taken for an alternating quantity to complete one cycle is called the period of the waveform.



Frequency: the number of cycles completed in one second is called the frequency of the waveform and measured n hertz, Hz.

$$1 hertz (Hz) = 1\ cycle\ per\ second$$

The Period and frequency of a sin wave can be related by the following equation:

$$T = \frac{1}{f}\ or\ f = \frac{1}{T}$$

Example1. Two sources have frequencies f_1 and f_2 respectively. If $f_2=2f_1$ and T_2 is 20ms, determine f1, f_2, and T_1?

$$f_2 = \frac{1}{T_2} = \frac{1}{20ms} = 50Hz$$

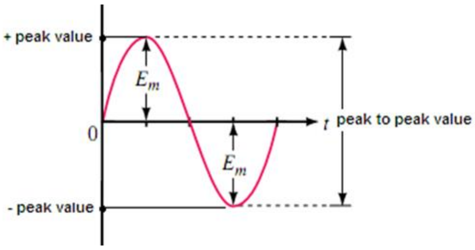
$$f_1 = \frac{f_2}{2} = \frac{50Hz}{2} = 25Hz$$

$$T_1 = \frac{1}{f_1} = \frac{1}{25Hz} = 40ms$$

Example 2: An alternating current completes 5 cycles in 8ms. What is its frequency?

Instantaneous value: the magnitude of a waveform at any instant of time; denoted by lower case letters (e_1, e2, i1, i2…)

Peak value: the maximum instantaneous value of a waveform as measured from the zero-volt level.

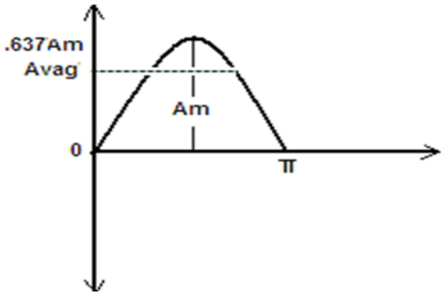


Peak-to-peak value: the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Average (mean) value:

Because a sine wave is symmetrical, its area below the horizontal axis is the same as its area above the axis; thus over a complete cycle the average value is zero. The average of half a sine wave, however, is not zero. Therefore the average value of sin wave is the average value measured over a half cycle.

$$Average(mean)value = \frac{area\ under\ the\ curve}{length\ of\ base}$$



$Area = \int_0^{\pi} A_m\, sin\alpha d\alpha$

$Area = \mathrm{A_m}[-cos\alpha]_0^{\pi} = -Am(cos\pi - cos0^0) = -A_m[-1-(+1)] = -A_m(-2)a = 2Am$

Since we know the area under the positive pulse, we can easily determine the average value the positive region of a sine wave:

$$Average(mean)value = \frac{Area\ under\ the\ curve}{base\ length}$$

$$= \frac{2A_m}{\pi}$$

Average value=0.637A_m

5.2. Sinusoidal and complex forcing functions

Equation of sinusoidal waveform

The basic mathematical formula for the sinusoidal waveform is:

$$e = E_m sin\alpha$$

Where ***e*** is instantaneous voltage, $\boldsymbol{E_m}$ is the maximum coil voltage and $\boldsymbol{\alpha}$ is the instantaneous angular position of the coil.

Angular Velocity (ω) the rate at which the generator coil rotates is called its angular velocity.

$$\omega = \frac{\alpha}{t}$$

Where, α is angular distance and t is time $\alpha = \omega t$

In practice, ω is usually expressed in radians per second, where radians and degrees are related by the identity. $2\pi\ radians = 360^0$

Relationship between ω, T and f

One cycle of sine wave may be represented as either $\alpha = 2\pi rads$ or t=Ts. Substituting these in to $\alpha = \omega t$ you get 2π=ωt.

$$\omega T = 2\pi(rad)$$

Thus, $\omega = \frac{2\pi}{T}$

Recall f=1/T Hz. Substituting this it to in the above equation you get

$$\omega = 2\pi f(rad/sec)$$

$$e = E_m sin\alpha \quad \text{but} \quad \alpha = \omega t$$

Combining these equations yields

$$e = E_m sin\omega t$$

Similarly, $i = I_m\, sin\omega t$

Substitute $\omega = 2\pi f$ to the above equation yields $e = E_m sin2\pi f t$

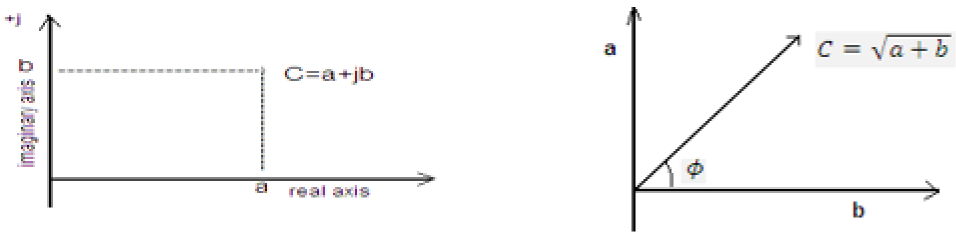
The general expression for an alternating voltage is $v = V_m sin(\omega t \pm \theta)$

Complex number review

A complex number is a number of the form $C = a + jb,$

where a and b are real numbers and $j = \sqrt{-1}$. The number a is called the real part of C and b is called its imaginary part.

Complex numbers may be represented geometrically, either in rectangular form or in polar form as points on a two-dimensional plane called the complex plane.



Rectangular form polar form

$$C = a + jb(rectangular\ form)$$

$$C = C < \phi(polar\ form)$$

Fig.5.3: complex number representation

Conversion between two forms

The two forms are related by the following equation,

Rectangular to polar

$$C = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}\frac{b}{a}$$

Polar to rectangular

$$a = C\cos\theta \qquad b = C\sin\theta$$

Mathematical operations with complex number

Let us first examine the symbol j associated with imaginary numbers by definition,

$j = \sqrt{-1}$, $j^2 = -1$, $\frac{1}{j} = -j$

Complex conjugate

The conjugate or complex conjugat of a complex number can be found by simply changing the sign of the imaginary part in the rectanguar form or by using the negative of the angle of the polar form.

The conjugate of $C = a + jb$ is $C = a - jb$

The conjugate of C= $C < \theta$ is C= $C < -\theta$

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately.

if $C_1 = a_1 + jb_1$ and $C_2 = a_2 + jb_2$ then,

$C_1 + C_2 = (a_1 + a_2) + j(b_1 + b_2)$

Subtaction

if $C_1 = a_1 + jb_1$ and $C_2 = a_2 + jb_2$

Then $C_1 - C_2 = (a_1 - a_2) + j(b_1 - b_2)$

Addition or subtraction can not be performed in polar form unless the complex numbers have the same angle θ.

Multiplication

To multiply two complex numbers in rectanguar form, mulitiply the real and imaginary parts of one in turn by the imaginary parts of the other.

$if\ C_1 = a_1 + jb_1$ and $C_2 = a_2 + jb_2$

Then $C_1 * C_2 = (a_1 + jb_1) * j(a_2 + jb_2)$

$C_1 * C_2 = (a_1a_2 - b_1b_2) + j(b_1a_2 + a_1b_2)$

To mulitiply two complex numbers in polar form,mulitiply magnitudes and add angles algebraically.

$if\ C_1 =< \theta_1$ and $C_2 =< \theta_2$

Then $C_1 * C_2 = C_1 * C_2 < \theta_1 + \theta_2$

Division

To divide two complex numbers in rectangular form, multiply the numerator and denominatior by the conjugate of the denomnator and resulting real and imaginary parts collected. That is, if

$if\ C_1 = a_1 + jb_1$ and $C_2 = a_2 + jb_2$

$$\frac{C_1}{C_2} = \frac{(a_1 + jb_1) * j(a_2 - jb_2)}{(a_2 + jb_2) * j(a_2 - jb_2)}$$

$$= \frac{(a_1a_2 + b_2) + j(b_1a_2 - a_1b_2)}{{a_2}^2 + {b_2}^2}$$

In polar form, division is accomplished by simply dividing the magnitde of the numerator by the magnitude of the denominator and subtracting the ange of the denominator from the numerator. $If\ C_1 =< \theta_1$ and $C_2 =< \theta_2$

$$\frac{C_1}{C_2} = \frac{C_1}{C_2} < \theta_1 - \theta_2$$

5.3. Phasor

The instantaneous levels of alternating current and voltage are vector quantities, since these levels are continuously changing, an AC waveform must be represented by rotating vector or phasor.

A phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoid ally varying quantities. The sinusoidal output voltage from the simple generator can be represented by the phasor diagram.

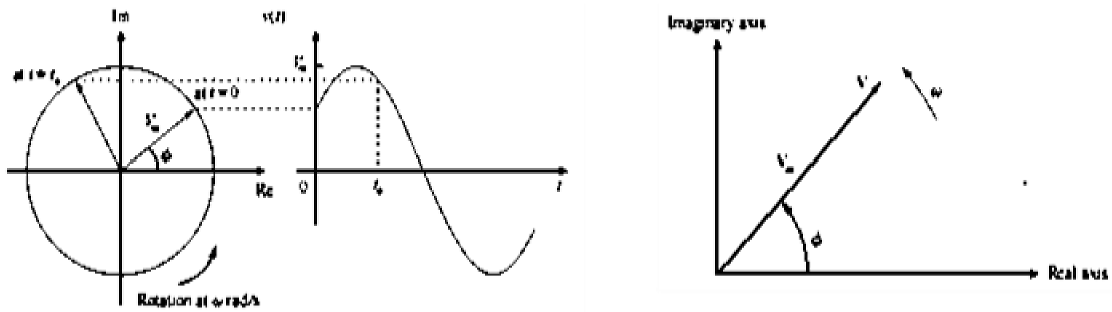


Fig.5. 4: Phasor reperesentation of AC waveform and phasor diagrm $\boldsymbol{v} = \boldsymbol{V_m} < \theta$

Voltages and currents with phase shifts

If a sine wave does not pass through zero at t=0 it has a phase shift. Waveforms may be shifted to the left or to the right. Mathematical equation for a waveform shifted to left

$$v = v_m \sin(\omega t + \theta)$$

Mathematical equation for waveform shifted to right

$$v = v_m \sin(\omega t - \theta)$$

Fig.5. 5 Phase shifted sine waves.

(a) $v = v_m \sin(\omega t + \theta)$ (b) $v = v_m \sin(\omega t - \theta)$

Sometimes voltages and currents are expressed in terms of $\cos\omega t$ rather than $\sin\omega t$. A cosine wave is a sine wave shifted by $+90^0$, or alternatively, a sine wave a cosine wave shifted by -90^0.

$$\cos(\omega t + \theta) = \sin(\omega t + \theta + 90^0)$$

$$\sin(\omega t + \theta) = \cos(\omega t + \theta - 90^0)$$

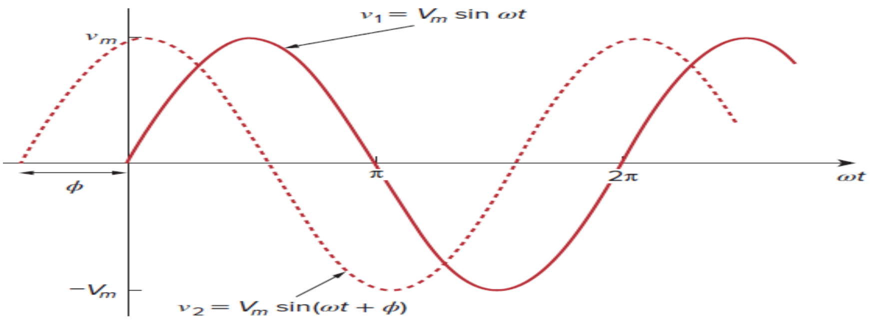
Phase difference

Phase difference refers to the angular displacement between different waveforms of the same frequency. If the angular displacement between two wave forms is 0^0 the waveforms are said to be **in phase**; otherwise, they are **out of phase**.

When describing a phase difference, select one waveform as reference. Other waveforms then lead, lag, or are in phase with this reference.

Let us examine the two sinusoids

$$v_1(t) = V_m \sin\omega t \quad \text{and} \quad v_2(t) = V_m \sin(\omega t + \phi)$$



- If $\phi = 0$, then v_1 and v_2 are said to be **in phase**.
- If $\phi \neq 0$, then v_1 and v_2 are said to be **Out of phase**.
- v_2 leads v_1 by ϕ or that v_1 lags v_2 by ϕ.

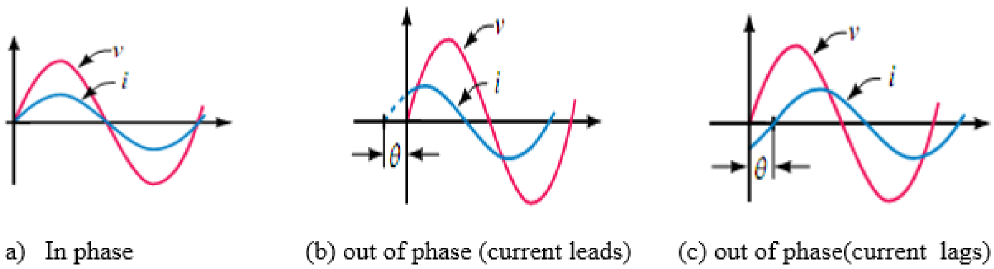


Fig.5. 6: Illustrating phase difference

Effective value (rms)

An effective value is an equivalent dc value: it tells us how many volts or Amps of dc that a time-varying waveform is equal to in terms of its ability to produce average power. A familiar example of such a value is the value of the voltage at the wall outlet in your home.

The effective value of a sine wave can be determined using the circuits of fig 5.6

- Consider a sinusoidal varying current **i(t)**. By definition, the effective value of ***i*** is that value of dc current that produces the same average power.
- Consider (b). Let the dc source be adjusted until its average power is the same as the average power in (a). the resulting dc current is then the effective value of the current of(a).to determine this value, determine the average power for both cases, then equate them.

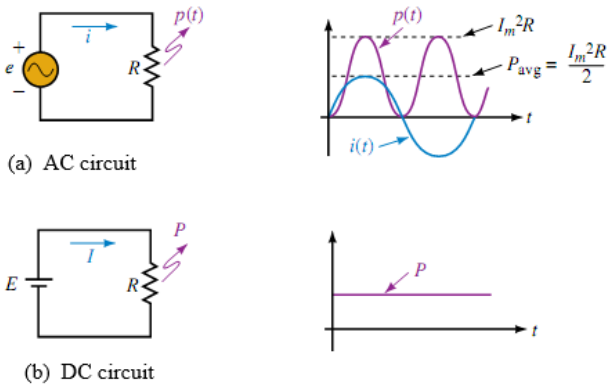


Fig.5.6: effective value of ***i***

Determining the effective value of a sinusoidal ac.

First, consider the dc case. Since current is constant, power is constant and average power is

$$P_{avg} = P = I^2R$$

Now consider the ac case. Power to the resistor at any value of time is P(t) i^2R, where *i* is the instantaneous value of current.

$$P_{ac=}(i_{ac})^2R = (I_m sin\omega t)^2R = (I_m^2 sin^2\omega t)R$$

But,

$$\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonomeric identity)}$$

Therefore

$$P_{ac} = I_m^2[\frac{1}{2}(1 - \cos 2\omega t)]R$$

And $$P_{ac} = \frac{Im^2}{2}R - \frac{Im^2}{2}R\cos 2\omega t$$

To get the average of P(t), note that the average of cos2ωt is zero and thus the last term of the above equation drops off leaving

$$P_{avg} = \frac{im^2}{2}R$$

Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{avg} = P_{dc}$$

$$\frac{Im^2}{2}R = Idc^2R \quad \text{and } I_m = \sqrt{2}\, I_{dc}$$

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.0707I_m$$

In word it can be stated that, the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its maximum value. In summery, $I_{dc} = I_{rms}(\text{effective value}) = 0.707I_m$ and $V_{rms} = 0.707V_m$

Example 2: An alternating voltage is given by $v = 282.8\sin 314t$ V. Find (a) the rms voltage (b) average voltage(c) the frequency and (d) the instantaneous value of voltage at t=4ms.

The general expression for an alternating voltage is: $v = V_m \sin(\omega t \pm \theta)$

a. $v_{rms} = 0.707 * V_m = 0.707 * 282.8V = 200V$

b. $\text{average voltage} = 0.637 * V_m = 0.637 * 282.8 = 180.14V$

c. $\text{angular velocity}, \omega = 314 \text{ rad/sec}, \text{ i.e } 2\pi f = 314$

$$f = \frac{314}{2\pi} = 50Hz$$

d. $\text{at t} = 4ms, v = 282.8\sin(314 * 4 * 10^{-3}) = 268.9\text{ V}$

Example 3: An alternating voltage is given by $v = 75\sin(200\pi t - 30^0)$ V.

Find (a) the amplitude, (b) the peak-to-peak value, (c) the rms value, (d) the periodic time, (e) the frequency, (f) the phase angle relative to $75\sin 200\pi t$

Comparing $v = 75\sin(200\pi t - 30^0)$ V with the general expression $v = V_m\sin(\omega t \pm \theta)$ gives:

a. $\text{amplitude, or peak value} = 75\text{ V}$
b. $\text{peak} - \text{to} - \text{peak value} = 2 * 75 = 150V$
c. $\text{the rms value} = 0.707 * \text{maximum value} = 0.707 * 75 = 150V$
d. $\text{angular velocity}, \omega = 200\pi \frac{rad}{s}$. Hence periodic time, $T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = 10ms$
e. $\text{frequency}, f = \frac{1}{T} = \frac{1}{0.01} = 100Hz$
f. $\text{phase angle } \theta = 30^0 \text{ lagging } 75\sin 200\pi t$

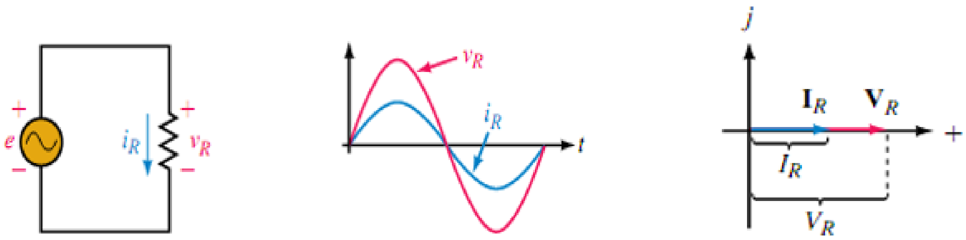
Exercise 1

1. If you double the rotational speed of an AC generator, what happens to the frequency and period of the waveform?
2. A 10Hz sinusoidal current has a value of 5A at t=25ms. What is its value at t=75ms?
3. Determine the phase relationship between voltage and current given by the following equations: $v = 50\sin(\omega t + 20^0)$ V and $i = 10\cos(\omega t - 45^0)$A
4. A sinusoidal current has a peak value of 30A and a frequency of 60Hz. At time t=0, the current is zero. Express the instantaneous current in the form $i = I_m \sin\omega t$.
5. An alternating voltage v has a periodic time of 20ms and a maximum value of 200V. When time t=0, v=-75V. Deduce a sinusoidal expression for v and sketch one cycle of the voltage showing important points.
6. An alternating voltage is represented by $v = 20\sin(300\pi t + 25^0)$ V. find (a) the maximum value (b) the frequency (c) the periodic time (d) rms value (e) average value.

5.4. Phasor representation for circuit elements

R,L and C circuit elements each have quite different electrical poroperties. Resistance, for example, opposes current, while inductance opposes changes in current and capacitance opposes change in voltage. These differences result in quite different voltage-current relationships.

Resistance in AC circuit(pure resistive circuit)



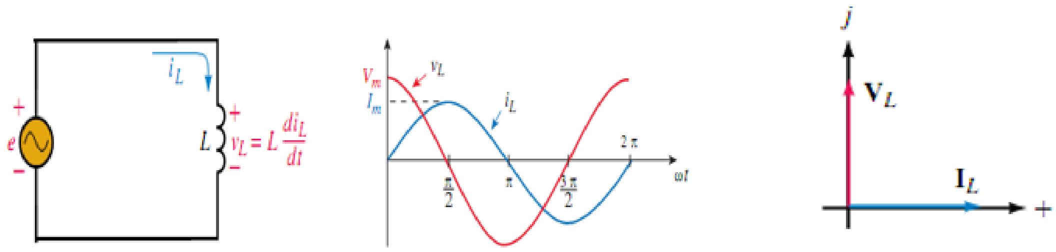
(a) Circuit diagram (b) waveform (c) phasor diagram

Fig.5. 8: Pure resistive circuit

In a pure resistive circuit current is in phase with voltage. The relation illustrated in fig 5.8 can be stated mathematical as:

$i_R = \frac{V_R}{R} = \frac{Vmsin\omega t}{R} = I_m sin\omega t$ where, $I_m = \frac{V_m}{R}$

Inductance in AC circuit



a. Circuit diagram (a) voltage and current wavforms (c) phasor diagram

Fig.5.9: pure inductive circuit

For an ideal inductor, volage V_L is prortional to the rate of change of current. Because of this, voltgae and current are not in phase as they are for a resistive circuit.

$$V_L = \frac{Ldi_L}{dt} = L\frac{d}{dt}(I_m sin\omega t) = \omega L I_m cos\omega t = V_m cos\omega t$$

Where $V_m = \omega L I_m$

Utilizing the trigonometric idntity $cos\omega t = \sin(\omega t + 90^0)$, you can write this as:

$$V_L = V_m \sin(\omega t + 90^0)$$

For a pure inductive circuit current lags volage by 90^{0}.

Inductive reactance(X_L)

From the above equation $V_m = \omega L I_m$. Thus $\frac{V_m}{I_m} = \omega L$

This ratio is defined as inductive reactance and is give the symbol X_L. since the ratio of volts to amps is ohms, reactance has units of ohms.

$$\text{Thus} \quad X_L = \frac{V_m}{I_m} = \omega L$$

$$XL = \omega L$$

But, $\omega = 2\pi f$

$$X_L(inductive\ reactance) = 2\pi fL \ \ (\Omega)$$

Reactane X_L represents the opposition that inductance presents to current for the sunusoidal as case. We now have everything that we need to solve simple inductive circuits with sinusoidal excitation, that is ,we know that current lags voltage by 90^0 and that their amplitudes are related by

$$I_m = \frac{V_m}{X_L}$$

$$V_m = I_m X_L$$

Example 4. A 0.5H inductor is conneceted across AC source. If the voltage across the inductor is $v = 100sin20t$ determine the inductive reactance and write the expression for the current.

$$X_L = \omega L = 2\pi fL$$

$$X_L = (20\,rad/s\ )(0.5H) = 10\Omega$$

$I_m = \frac{V_m}{X_L} = \frac{100V}{10\Omega} = 10A$ we know that i lags v by 90^0. Therefore,

$i = 10\sin\,(20t - 90^0)$ A

Capacitance in Ac circuit(pure capacitve circuit)

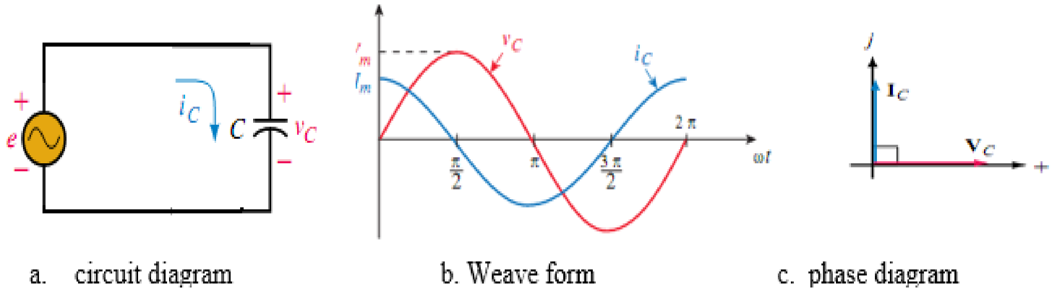


Fig.5.10: pure capacitive circuit

For capacitance, current is proportional to the rate for change of voltage, i.e.

$$i_c = c\frac{dV_C}{dt} = c\frac{d}{dt}(V_m sin\omega t) = \omega cV_m cos\omega t$$

$$i_c = I_m cos\omega t \qquad \text{where } I_m = \omega cV_m$$

Using the appropriate trigomometric identity($\cos(\omega t) = \sin\,(\omega t + 90^0)$), the above equation can be written as

$$i_c = I_m \sin\,(\omega t + 90^0)$$

For a purely capacitive circuit, current leads voltage by 90^0.

Capactive ractance(X_C)

Now consider the relationship between maximum capacitor voltage and current magnitudes.

$$I_m = \omega CV_m$$

Then,

$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

The ratio of V_m to I_m is defined as capacitive reactance and is given the symbol X_c. that is,

$$X_c = \frac{V_m}{I_m} = \frac{1}{\omega C} \ (\Omega)$$

$$X_c = \frac{1}{\omega C} \qquad \text{But, } \omega = 2\pi f$$

Thus, $X_C(capactive\ reactance) = \frac{1}{2\Pi fc}$ (Ω)

Reactance X_C represents the opposition that capacitance presents to current for the sinusoidal ac case. We now have everything that we need to solve simple capacitive circuits with sinusoidal excitation. i.e., we know that current leads voltage by 90^0 and that

$$I_m = \frac{V_m}{X_C} \text{ and } V_m = I_m X_C$$

Example 4.A 1µFcapacitor is conneceted across AC source. If the voltage across the capacitor is $v = 30sin400t$ determine the capacitive reactance and write the expression for the current.

$$X_C = \frac{1}{\omega C} = \frac{10^6}{400}\ \Omega = 2500\Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30V}{2500\Omega} = 12mA$$

and we know that for acapcitor i leads v by 90^0. Therefore,

$$i = 12\sin(400t + 90^0)\, mA \text{ A}$$

5.5. Impedance and admittance and Phasor diagrams

AC series circuit

When we examined dc circuits we saw that the current everywhere in series circuit is always constant. This same applies when we have series elements with ac sources. Further, we had seen that the total resistance of a dc series circuit consisting of n resistors was determined as:

$$R_T = R_1 + R_2 \dots \dots .. R_n$$

When working with ac circuits we no longer work with only resistance but also with capacitive and inductive reactance.

Impedance

Impedance is a term used to collectively determine how the resistance, capacitance, and inductance “impede” the current in ac circuit. The symbol for impedance is the letter Z and the unit is the ohm (Ω).

Because impedance may be made up of any combination of **resistance** and **reactance**, it is written as a vector quantity Z, the polar form impedance is written as:

$$Z = Z < \phi \ (\Omega)$$

The value Z is the magnitude(in ohms) of the impedance vector Z and is deteremined as:

$$Z = \sqrt{R^2 + X^2} \ \ (\Omega)$$

The corresponding angle of the impedance vector is determined as:

$$\phi = \pm \tan^{-1}(\frac{X}{R})$$

The rectangular form of impedance is written as:

$$\boldsymbol{Z = R \pm jX,}$$

Where R is resistance and X is reactance (XL or XC).

If we are given the polar form of the impedance, then we may determine the equivalent rectangular expression from as:

$$R = Zcos\phi \quad \text{and} \quad X = Zsin\phi$$

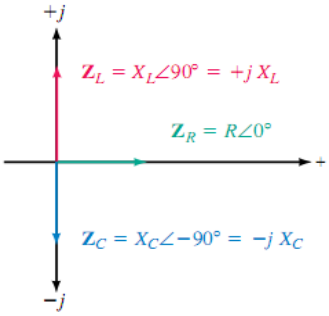


Fig.5.11: Impedance diagram

Resistive impedance Z_R is a vector having a magnitude of R along the positive real axis; Inductive impedance Z_L is a vector having a magnitude of X_L along the positive imaginary axis, while the capacitive impedance Z_c is a vector having a magnitude of X_c along the negative imaginary axis. Mathematically, each of the vector impedance is written as follows

$$Z_R = R < 0^0 = R + j0 = R$$

$$Z_L = X_L < 90^0 = 0 + jX_L = jX_L$$

$$Z_C = X_C < -90^0 = 0 - jX_C = -jXc$$

R-L circuit

RL circuit is the combination of resistive and inductive load.

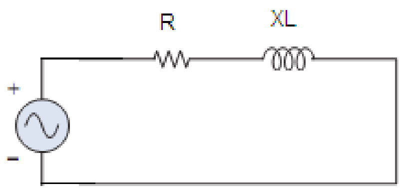
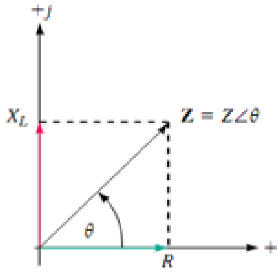


Fig.5.12: RL circuit

In RL circuit the total impedance Z is

$$Z = R + jX_L or \quad Z = Z < \tan^{-1}(\frac{X_L}{R})$$



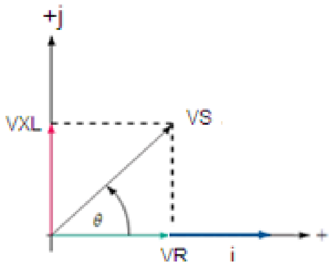


Fig.5.13: (a) Impedance diagram (b) Phasor diagram

Voltage across resistor(R) and indictor (L) can be determined as

$$V_R = i * Z_R$$

$$V_L = i * Z_L = i * X_L$$

Thus the total voltage (supply voltage, V_s)

$$V_s = V_R + jV_L$$

The total circuit current (i):

$$\boldsymbol{i = \frac{V_s}{Z} = \frac{V_R+jV_L}{R+jX_L} or} \quad i = \frac{V_s<\phi_1}{Z<\phi_2} = \frac{V_s}{Z} < \phi_1 - \phi_2$$

Example 5: A 4Ω resistor and a 9.55mH inductor are connected in series with 240 V, 50 Hz AC source. Calculate (a) inductive reactance (b) the impedance, (c) the total current, and (d) draw impedance and phasor diagram.

a. $inductive\ reactance, X_L = 2\pi f L = 2\pi(50)(9.55 * 10^{-3}) = 3\Omega$

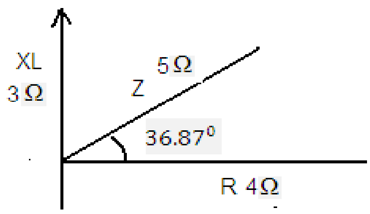
b. $impedance, Z = \sqrt{R^2 + {X_L}^2} = \sqrt{4^2 + 3^2} = 5\Omega$

c. $current, i = \frac{v}{z} = \frac{240V}{5\Omega} = 48A$

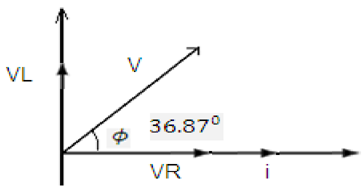
d. $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{3}{4} = 36.87^0\ lagging$

$V_R = i * R = 48 * 4 = 192 < 0\ V$

$V_L = i * X_L = 48 * 3 = 144V\ but\ \ V_L = 144 < 90^0\ V$



Impedance diagram



Phasor diagram

Therefore, in the above example current lags voltage by 36.87^0

R-C circuit

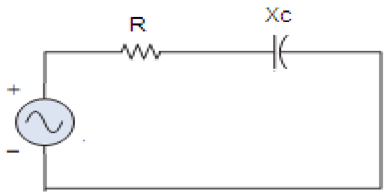
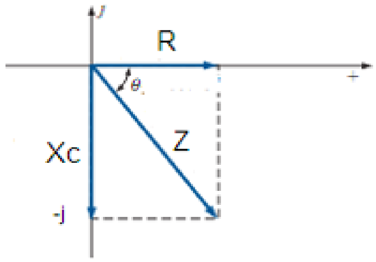


Fig.4. 2: RC circuit

In RC circuit the total impedance Z is written as:

$$Z = R - jX_C \quad or \quad Z = Z < \tan^{-1}(\frac{X_C}{R})$$



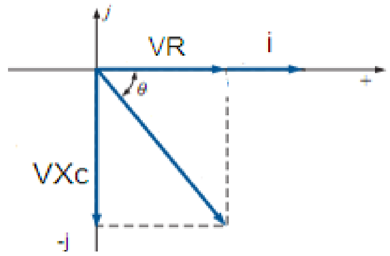


Fig.5.14: (a) Impedance diagram (b) Phasor diagram

Voltage across resistor(R) and capacitor (C) can be determined as

$$V_R = i * Z_R$$

$$V_C = i * Z_C = i * X_C$$

Thus the total voltage (supply voltage, V_s)

$$V_s = V_R - jV_C$$

The total circuit current (I_T):

$$i = \frac{V_s}{Z} = \frac{V_R - jV_C}{R - jX_C} \quad or$$

$$i = \frac{V_s < \phi_1}{Z < -\phi_2} = \frac{V_s}{Z} < \phi_1 + \phi_2$$

Example 6: A resistor of 25Ω is connected in series with a capacitor of 45µF. calculate (a) the impedance, (b) the current taken from a 240,50Hz supply. Find also the phase angle between the supply voltage and the current.

- Capacitive reactance, $X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(45*10^{-6})} = 70.74\ \Omega$
- Impedance $Z = \sqrt{R^2 + {X_C}^2} = \sqrt{25^2 + 70.74^2} = 75.03\Omega$
- Current, $i = \frac{V}{Z} = \frac{240V}{75.03\Omega} = 3.2A$

Phase angle between the supply voltage and current $\phi = \tan^{-1}\frac{X_C}{R} = \frac{70.74}{25} = 70.54^0$ leading. i.e., current leads supply voltage by 70.54^0

Series RLC circuit

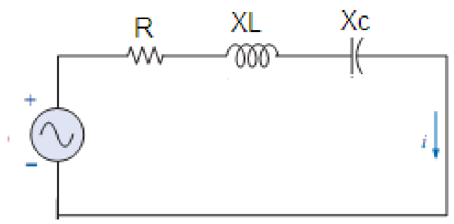


Fig.5. 3: RLC circuit

In RLC circuit the total impedance Z written as

$Z_T = Z_R + Z_L + Z_C = R + jX_L - jX_C$

$Z_T = R + j(X_L - X_C)$

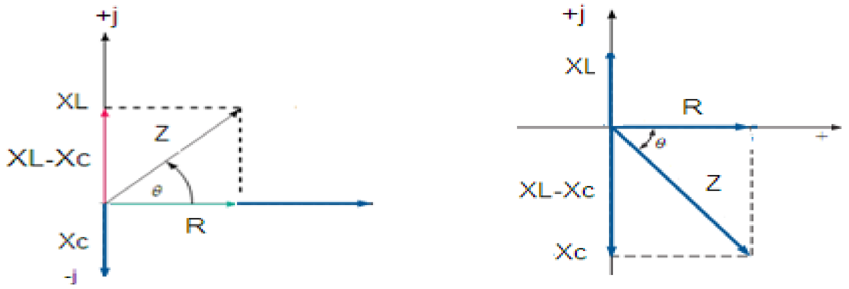


Fig.5. 4: (a) Impedance diagram ($for\ X_L > X_C$)   (b) impedance diagram (for$X_L < X_C$)

Voltage across each circuit element will be:

$V_R = i * R$,$V_C = i * X_C$ and $V_L = i * X_L$

Where $i = \frac{V_s}{Z}$ And $V_s = V_R + j(V_L - V_C)$

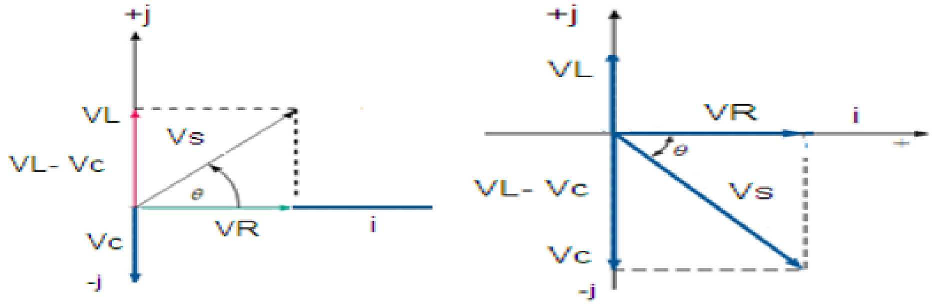


Fig.5. 5: Fig4.15 (a) Phasor diagram (for $X_L > X_C$) (b) Phasor diagram (for$X_L < X_C$)

Example 7: A 5Ω resistor, 120mH inductor and 100μF capacitor are connected in series to a 300V, 50Hz AC supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the circuit elements, and (d) draw the phasor and impedance diagram.

$$X_L = 2\pi f L = 2\pi(50)(120 * 10^{-3}) = 37.70\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(100 * 10^{-6})} = 31.83\Omega$$

Since X_L is greater than X_C the circuit is inductive

$$X_L - X_C = 37.7 - 31.83 = 5.87\Omega$$

$$impedance\ (Z) = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{5^2 + 5.87^2} = 7.71\Omega$$

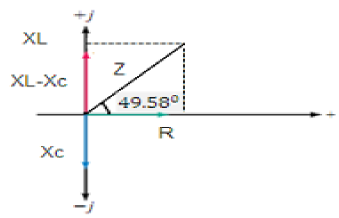
a. $current\ (i) = \frac{v}{z} = \frac{300}{7.71} = 38.91A$

b. $phase\ angle\ \phi = \tan^{-1}(\frac{X_L - X_C}{R}) = \tan^{-1}(\frac{5.87}{5}) = 49.58^0$

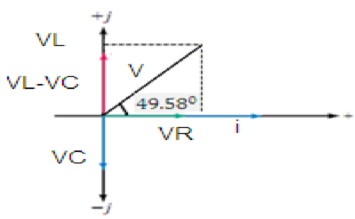
c. $V_R = i * R = 38.91A * 5\Omega = 194.55V$

$$V_L = i * X_L = 38.91 * 37.7 = 1466.9V$$

$$V_C = i * X_C = 38.91 * 31.83\Omega = 1238.5V$$



Impedance triangle



Phasor diagram

Parallel RLC circuit

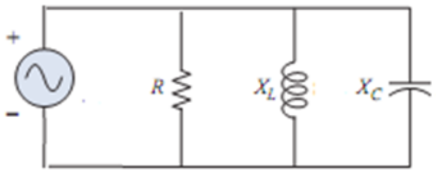


Fig.5. 6: RLC circuit

Admittance

Admittance is defined as a vector quantity which is the reciprocal of the impedance Z. Mathematically, admittance is expressed as:

$Y_T = \frac{1}{Z_T} = \frac{1}{Z_{T<\theta}} = \left(\frac{1}{Z_T}\right) < -\theta = Y_T < -\theta$. The unit of admittance is the Siemens(S).

The admittance of resistor R is called conductance and is given a symbol Y_R.

$$Y_R = \frac{1}{R<0^0} = G < 0^0 = G + j0$$

The admittance of a purely reactive component X is called susceptance of the component and is assigned the symbol B. The unit for susceptance is Siemens (S).

In order to distinguish between inductive susceptance and capacitive susceptance, we use the subscripts L and C respectively.

$$Y_L = \frac{1}{X_L < 90^0} = \frac{1}{X_L} < -90^0 = B_L < -90^0 = 0 - jB_L$$

$$Y_C = \frac{1}{X_C < -90^0} = \frac{1}{X_C} < 90^0 = B_C < 90^0 = 0 + jB_C$$

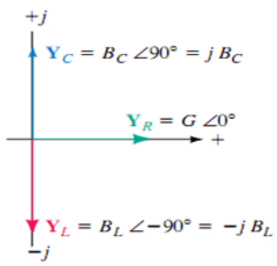


Fig 5.19: Admittance diagram

The total impedance (Z_T) in parallel RLC circuit can be calculated as:

$$\frac{1}{Z_T} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Y_T = Y_R + Y_L + Y_C$$

$$Z_T = \frac{1}{Y_T}$$

The total current (I_T)

$$I_T = \frac{V_s}{Z_T} = V_s * Y_T$$

Exercise 2

1. Calculate the current taken by 23μF capacitor when connected to a 240 V, 50 Hz supply.

2. A coil has an inductance of 40mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to (a) a240 V 50Hz supply and (b) a 100 V, 1 kHz supply.
3. A coil of inductance 300mH and negligible resistance is connected in series with 100Ω resistor to a 250V, 50Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) voltage across each component and (e) the circuit phase phase angle.
4. A capacitor C is connected in series with a 40Ω resistor across a supply of frequency 60Hz. A current of 3A flows and circuit impedance is 50Ω. Calculate (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) voltage across the resistor and capacitor, and (e) draw phasor and impedance diagram.
5. A 40μF capacitor in series with a coil of resistance 8Ω and inductance 80mH is connected to a 200V, 100Hz supply. Calculate (a) the circuit impedance, (b) the current flowing, (c) the phase angle between voltage and current, (d) the voltage across the coil, and the resistor, (e) the voltage across the capacitor, and (f) draw the phasor and impedance diagram.

5.6. AC circuit analysis techniques

In this section, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits if the excited with AC source.

Steps to Analyze circuits

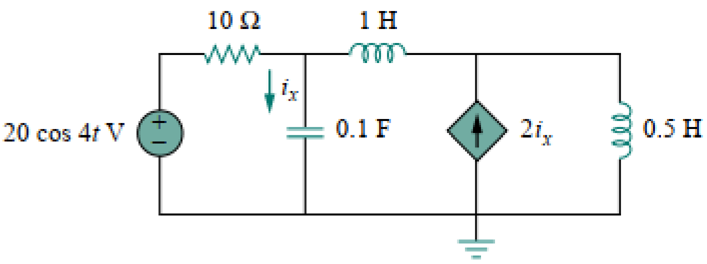
1. Transform the circuit to the phasor or frequency domain.

2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).

3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved. Having read Chapter 3, we are adept at handling step 3.

Example Find *ix* in the circuit of Fig. bellow using nodal analysis.



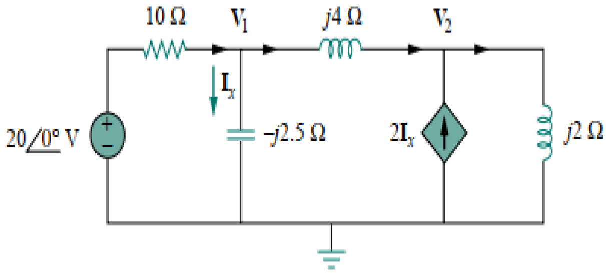
We first convert the circuit to the frequency domain:

$$20\cos 4t \Longrightarrow 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$

$$1 \text{ H} \Longrightarrow j\omega L = j4$$

$$0.5 \text{ H} \Longrightarrow j\omega L = j2$$

$$0.1 \text{ F} \Longrightarrow \frac{1}{j\omega C} = -j2.5$$



Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad (1$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

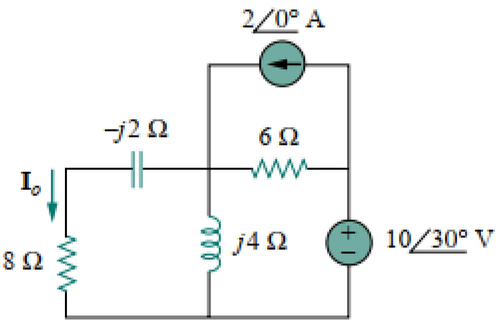
$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

Exercise:

1. Determine current **I***o* in the circuit of Fig. bellow using mesh analysis.



2. Find *vo* in the circuit in Fig. bellow using the superposition theorem.